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ALGORITHM OF STRATIGRAPHIC CORRELATION

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Defects in existing methods of correlation depend on the lack of mathematical enunciation of the correlation problem and an unsatisfactory state of the concept system employed for its solving.

When enunciating a problem it is not clear usually, what logical operations are to be conducted with initial data, what are these data and how the solution will be estimated. In case two contradictory solutions are obtained, it is not clear, what unambiguous objective procedure will help us to select the right one. The concepts used for solving the problem answer neither the requirements of gnosiology and logic nor those of pragmatics. It is not known, for example, what is meant by "one and the same stratigraphic level" and "different stratigraphic levels," "narrow" and "broad stratigraphic ranges" and so on. We know nothing of the way these concepts were inferred from observations. Moreover, we do not know what observations must serve as empiric basis for the inference of these and other stratigraphic concepts. By means of analysis numerous contradictions and logic circles are found in possible logico-mathematical interpretations of generally accepted intuitive inferences. Besides, we do not know why just these concepts, being defined precisely like this but not otherwise, must be used for the optimum solving of the problem.

We shall try to enunciate the problem of correlation, to define the concepts used when solving it and to work out the procedure of solving. We shall strive at the maximum usage or methods and concepts generally employed in stratigraphy. But we intend to make them suitable for treatment at computers and answering the requirements of gnosiology, logic, and pragmatics.

ENUNCIATION OF THE CORRELATION PROBLEM

At first we are to clear up the position of the correlation problem among other stratigraphic problems. The following succession is generally accepted: description of local sections—their dismembering—correlation—building of boundaries of correlated bodies (Voronin *et al.*, 1971, 1972; *Stratigraphy and Mathematics* 1974). The last problem results in a geologic map. To the sphere of stratigraphy belong three first problems.

Description of local sections may be interpreted as studying of the observed properties distribution in one-dimensional space.

Dismembering may be interpreted as division of one-dimensional space into geologic bodies. A geologic body may be defined as a part of space; its all points have the properties of one class in a given classification. The classification used in dismembering must be given beforehand; it must answer the following requirement; no meaning of any property must be found in two classes or be left aside of the classes. According to the principle of specialization (Kosygin, 1970) we carry out dismembering using all the classifications separately. Thus in accordance with a lithological classification we divide a section on beds of sandstones, argilites, limestones, and so on; according to a geophysical one (geoelectric)—on intervals with high, middle, and low resistance, and so on. Boundaries are built in those points of space, where meanings of properties become bordering between classes of the classification used.

It is inexpedient to divide space into bodies according to all the complex of available properties as it is done when geologic problems are being solved by a statistical approach (Rodionov, 1968). We are practically never interested in all the properties of bodies under analysis. A coal geologist is sure to be unsatisfied by a stratum traced by all the properties except coal presence. Among the available properties one should distinguish between those which are prescribed (they are interesting to us from a practical point of view) and subsidiary ones (they permit to establish distribution of those which are prescribed). We shall not proceed from a hypothesis about accidental relations between different properties. On the contrary, our purpose is to find regularities in relations between bodies discriminated on the basis of different properties, so as to use them when solving stratigraphic problems.

Let us state that dismembering should be carried out individually only on the basis of different prescribed classifications. Subsidiary classifications can be used for correlation without solving the problem of dismembering. Such saving of the computering procedures can be attained by introducing special concepts defined in terms of the purpose criterion.

The term correlation is understood differently. There exist advocates of both narrow and broad understanding. The former consider correlation only as synchronization, i.e. identification by geologic age. The last include here synchronization and establishment of faunal, lithological, and facies correspondences (Krumbein and Sloss, 1960). We accept the second point of view.

Methods of stratigraphic synchronization are numerous. If several of them are employed for synchronization of one and the same pair of objects contradictory conclusions can be often reached. But to estimate the result objectively and to choose the right one is impossible. It is due to the fact that we had no definition of geological simultaneity unambiguously inferred from the observations (Salin, 1974). Such a definition appeared only recently (Kosygin *et al.*, 1974). It can be shown, that its usage makes synchronization more feasible in comparison to other correlations.

We accept the following enunciation for the problem of untemporary correlation (Voronin *et al.*, 1971, 1972):

Correlation is an answer to the question whether two one-dimensional geologic bodies of different sections (i.e. stratigraphic units of these sections) belong to one and the same two- or three-dimensional body, which crosses both sections, or whether they belong to different two- or three-dimensional bodies.

Solution of the correlation problem in accepted enunciation allows an objective estimation: by means of immediate observation in space between sections one can establish whether a body is present in predicted place or not.

Problems of establishment of a lithological continuity, lateral continuity (Rodgers, 1959; Lowman, 1953) and synonymy of coal strata can be enunciated like this. The essence of the problem is in tracing of real geologic bodies from one section to another, regardless of their boundaries simultaneity.

Many problems, usually referred to as problems of dismembering can be enunciated in such a way. In regions composed by thick monotonous terrigene or volcanic rock masses, or, on the contrary, by rock masses with various lithological composition repeating themselves completely in any parts, it is difficult to discriminate big lithological bodies which could be traced from one section to another. Difficulties appear here not because of dismembering: in every local section taken separately a geologist can easily discriminate distinct bodies and packet. Difficulty lies exactly in separation of rock masses into units equal for all sections but differing from overlying and underlying units, i.e. difficulties appear exactly because of the correlation. This procedure can be called dismembering predetermined by the correlation possibilities. The requirements to "suites" in soviet geology and to "map units" in american geology are justified. They are as follows: In local sections those units must be discriminated, which can be traced throughout the whole of a region.

Two relationships need definition in enunciation of the correlation problem. They are "to be parts of one and the same body" ("to belong to one and the same body") and "to overlie."

Definition of the former can be taken from topology. In this science only those parts are considered as belonging to one and the same body, which can be connected by a continuous line, all the points of this line belong to that body. Such a relationship has a name of compendency in topology In american stratigraphy there is a close concept of a lithological continuity.

According to previously accepted condition, only those points belong to one and the same body, which are characterized by the properties of only one class in the employed fixed classification. Hence, it can be said that to one and the same body belong only the parts, which can be connected by a continuous line, every point of which is characterized by the properties of some one class. It is then supposed that the parts, which can be connected must possess the properties of the same class.

At first we shall define the relation "over" for a pair of points a and b:

Point a overlies point b if and only if two of them are situated on the same vertical line and an absolute mark of point a is higher than that of point b.

It should be stressed that if points are not on the same vertical line, then the relation "over" has no sense; none "higher" hypsometric position allows conclusions about stratigraphic relations between two points. Besides, a vertical line in the Earth's coordinates, plumb line is meant here, but not a "stratigraphic vertical line," "stratigraphic normal." A "stratigraphic normal" is built as a perpendecular to a bedding plane, which in its turn results from the boundaries building. The boundaries building problem is being solved after the correlation problem; therefore, inclusion of a "stratigraphic normal" into a definition destined for the correlation purposes leads to a logical circle.

The relation "to overlie" for geologic bodies we define through the relations of the points belonging to them:

A body A overlies a body B if and only if any point of a body A overlies any point of a body B, situated on the same vertical line.

MEANS OF CORRELATION

To solve the correlation problem in accepted enunciation is possible if in the sections under comparison there are at least single bodies of one and the same class of the fixed classification used for the sections dismembering. If such bodies are more than one the solution of the problem is not unique. In order to reduce variants of the solution, limitations are necessary.

Yu. A. Voronin (1967) suggests the following limitation: ". . . beds cannot cross" (p. 76); D A. Rodionov's condition (1968) is: ". . . if in two sections under comparison there is a pair of stratigraphic units, which can be united, then the correlation of overlying units in one section with underlying units in the other one has no sense" (p. 56). Voronin-Rodionov's condition greatly reduces variants of solution. But still it is not enough usually for the reduction of variants to an acceptable minimum.

Great possibilities are provided by regularities of the relations between bodies discriminated on the basis of different classifications. For correlation of bodies discriminated on the basis is of one classification with the help of bodies discriminated on the basis of another one, only those subsidiary bodies are useful, which give boundaries not crossing the prescribed ones. If we correlate, for example, sandy rock mass (Figure 1) with the fauna α in the section A and similar rock mass with the fauna α in the section B then we'll make a mistake.

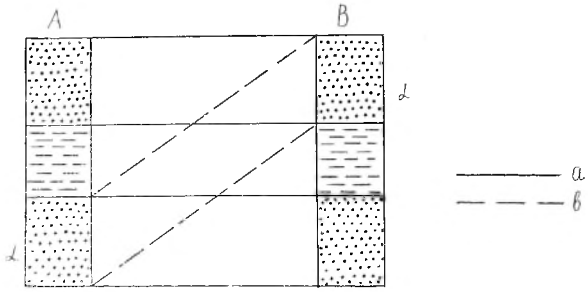

FIG. 1. A possible example when lithological boundaries cross paleonthological ones.
a—boundaries of lithological bodies
b—boundaries of the fauna α distribution.

For selection among a number of bodies discriminated on the basis of different classifications, those, boundaries of which do not cross, it is necessary to work out algorithm.

Several concepts necessary for working out algorithm should be introduced.

If all the bodies with a feature α overlie any body with a feature β, then such features are called stratifying. (By a feature we'll call the meaning of a property, belonging to one class in a fixed classification.)

However, the introduced definition is convenient only for such features, on the basis of which continuous geologic bodies may be discriminated, i.e. for lithological, geochemical, and geophysical features. The fossils, which are paleonthological features, are localised in space in such a way that their presence most often does not allow to discriminate a continuous geologic body. In order to establish continuity it is necessary to have in the immediate vicinity of some point in a given body (i.e. a point with a feature of a given class) another point in the same body (with a feature of the same class), in the vicinity of this other point—the third point, etc. As belonging to one and the same body are considered those parts between which a continuous way from one vicinity to another is possible. For lithological, geophysical, and geochemical features constituting the field (space, every point of which is characterised by the value of some property) radius of the vicinity may be done infinitesimal. For paleonthological features radius may be only finite. If radius is for example ten metres, then, on the one hand, points, distances between which exceed ten metres, will be unconnected. As a result many parts of a body will dissociate on to disconnected individual bodies. On the other hand, a body so discriminated cannot be narrower than ten metres and this is not acceptable for many detailed correlations. Radius reduction leads to further dissociation of a united body into individual small ones. If radius grows, minimum dimensions of a body become greater.

But even for the properties constituting the field, problem of division into bodies is very complex. Observations are very often discrete, unexplored intervals are very often situated between points of observation. When conducting dismembering, we have to make suppositions about the value of a property in unobserved intervals, i.e. to solve the problem of interpolation. Problem of interpolation cannot be solved unambiguously without some assumption. Thus one has to seek for such assumptions, to formulate them, to arrange them into an uncontradictory system and so on. Difficulties are numerous. To treat them only as mathematical and having nothing to do with geology is not right. For example, in a local section the Oligicene fossil is found in one locality and the Miocene fossil occurs one hundred metres higher. Building of boundaries between Oligocene and Miocene according to such data is a typical interpolation. It is well known, what discussions and disagreements result from such an interpolation.

Now let us turn to the purposes criteria. If features interesting to us are given, then there is no need to discuss the necessity of dismembering. Space division into bodies according to these features and establishment of relations between them, i.e. structure—it is the purpose of investigation. But if the features, among which we seek for stratifying ones, are subsidiary it is possible to use them for correlation not solving the problem of dismembering. Such an approach contradicts to practice. In our early works (together with Yu. A. Voronin *et al.*, 1971, 1972) the relations between features were also defined through relations between bodies. In order to avoid solving of dismembering problem let us reformulate the definition:

If all the points with a feature α overlie any point with a feature β, then such features are called stratifying.

In this case a set of points with paleonthological features will be discrete and sets of points with lithological, geochemical, and geophysical features can group into continuous subsets, i.e. bodies.

By a system of stratifying features we call such a set of features in which every one of them is stratifying in relation to any other.

In a system of stratifying features all the boundaries do not cross with one another. Indeed, a body with a feature α underlying a body with a feature β in the same section, for crossing it must appear in another section above it, coincide with it or be included or inclusive, but this contradicts to the condition: feature α is stratifying in relation to β.

Having required that the boundaries of correlated bodies should not cross the boundaries built on the basis of stratifying features it is possible to limit variants of correlation greatly. The richer system of stratifying features the less number of variants will be. If we do not limit the number of features for choosing stratifying ones among them, the interval of correlation ambiguity may become so narrow that divergence of variants within it can be disregard. At last this can result in a unique solution of the correlation problem.

CONSTRUCTION OF STRATIFYING FEATURES SYSTEM

Concrete described sections, where between interesting to us features stratigraphic relations were immediately observed, are considered as the only sources of data. The relation between α and β is marked by a symbol R_1 if features α and β are stratifying in reference to one another, and α is over β. A symbol R_2 indicates the contrary relation, i.e. "lower." And finally, if features α and β are not stratifying in a given local section, a symbol R_3 indicates the relation between them.

Let us build a square matrix. All the classes of the all employed classifications let be lines and columns of this matrix: $\alpha_1, \alpha_2, \ldots, \alpha_i, \ldots, \alpha_n$; $\beta_1, \ldots, \beta_j, \ldots, \beta_m$; etc., i.e. all the rocks, all the species of fossils found in the local sections all the classes of any elements contents (using geochemical criteria of correlation) and so on.

Let us begin to fill the matrix with symbols of relations observed in some one of the local sections. A symbol R_1 is put into a cell on the crossing of a line α_i and a column β_j if in this section α_i is over β_j, symbol R_2 if α_i is under β_j, or symbol R_3 if α_i and β_j are not stratifying in reference to one another. Similarly we put into the cells of the matrix relations observed in other sections.

The following situations are possible when filling the matrix:

1) Several equal symbols are put into a cell—relations between bodies possessing a feature α_i and bodies possessing a feature β_j are the same in all the sections, where these bodies are found together.

2) Different symbols are put into a cell—relations between bodies with a feature α_i and bodies with a

feature β_j differ in the sections, where these bodies are found together.

3) Finally, some cells are left not filled at all—bodies with features under consideration are not found together in none of the sections.

The third group of cells provides no empiric material about crossing or not crossing of the boundaries. The fact that the boundaries cross is evident for the second group. For example, a body with a feature α_i underlying a body β_j in one section must cross it in order to appear over it in another section. Such features cannot be stratifying because it is clear that only some points α_i overlie β_j (and vice versa). Consequently, a symbol R_3 must be put into such a cell.

The first group of cells is the most interesting. Cells with symbols R_3 do not help us in reducing variants of correlation because some bodies with a feature α_i overlie bodies β_j, others underlie them, and to distinct the former from the latter according to available data is not possible. Cells into which only symbols R_1 or R_2 are put allow conclusions about uncrossing boundaries of bodies with those features. Such features may be considered as stratifying.

According to the matrix a construction of several variants of stratifying systems is possible. We start to build a stratifying succession from the bottom. It is understandable, that optimum features for being used as a basis of this succession would be those, which occur under the rest ones, i.e. features to which correspond lines with symbols R_2 only. But it may so happen that these features are absent. Let us weaken the requirement and seek for features placed not over of any other feature. Lines with no symbols R_1 will correspond to them. But even in this case a situation is possible, when such features are absent. And once again we have to weaken the requirement and seek for lines with minimum of R_1.

If features satisfying the requirement of min R_1 are several we shall build successions for each of them separately. Suppose α_i is such a feature. Let us find a feature β_j having the relation R_1 only with α_i or with other already chosen features serving as a basis of other successions. Later we write out a feature γ_s having the relation R_1 with β_j or simultaneously with β_j and α_i and having no relations R_2 or R_3 with no one member of already built given succession, etc.

From the all built successions we are to choose the only one, because the usage of several successions at one and the same time for correlation will cause contradictions in the correlation.

CHOICE OF THE UNIQUE VARIANT OF A STRATIFYING SUCCESSION

We shall seek for the most useful system. It is obvious, that the most useful is such a system which provides the most detailed dismembering and the widest correlation.

The most wide correlation is ensured by the most widely distributed features. The wider a feature is distributed throughout the territory under examination, the more often it will be found together with other features, the more cells will be filled in the line of a matrix corresponding to it, the less vacant cells will be left. On the other hand, if a feature is distributed in every concrete section from the bottom to the top, then cells with symbols R_3 will correspond to it. It is evident that the less vacant cells and cells with a symbol R_3 will be in a line of a matrix, the more useful will be the feature for dismembering and correlation. The minimum "vacant cells + cells with a symbol R_3" is equal to the maximum "cells with a symbol R_1 + cells with a symbol R_2."

As we have to compare not individual features but their systems the requirement max $(R_1 + R_2)$ is made to the system as a whole. This variant seems optimum according to other reasons also.

1) Later a stratifying succession will be used for introduction of the features equivalence intervals (stratigraphic ranges) not included into the succession. The equivalence interval of any features is suggested to establish through its order relations with the features of a stratifying succession. It is understandable that with a greater number of, features not included into a succession, the features of a succession will have order relations R_1 or R_2, then for a greater number of them it will be possible to establish stratigraphic ranges.

2) Verification of the built stratifying succession according to new factual data can result in a conversion of some of its members, previously considered as stratifying, into not stratifying ones. Because of this, they must be excluded from the succession. A succession will be broken into two or three isolated parts. If those parts were connected only by excluded elements and no element of one part had observed order relations with any element of another part, to unite them would appear impossible. It is evident that the greater is a number of other elements of the succession, with which every element of the succession is connected,then the more chances exist for uniting the succession in case of any of its breaks, and the more stable it will be in

case of any change dependent on receiving of new factual material.

INTRODUCTION OF THE TRANSITIVITY PROPERTY

The transitivity condition is introduced for all the elements of a chosen succession:

if α is over β, and β is over γ, then α is over γ.

The necessity of this condition results from the fact that not for every element of the succession there exist observations about its relations with any other element. For example, γ_s may be never found in concrete sections together with ψ_k. For these two features we infer relations from observations for the relations of γ_s to σ_p and σ_p to ψ_k. Because the system is chosen in such a way that any pair of neighbouring member is connected with only one order relation and the features in the succession do not repeat one another (in this case they would have been not stratifying), then contradictions in inference of the relations of γ_s to ψ_k through any third σ_p, δ_t, . . . , etc., are excluded.

Logically inferred unobserved order relations need explanations. According to a definition, the relations "over" or "under" have sense only for the points situated on one vertical line. But the bodies with features γ_s and ψ_k are not situated on one vertical line. Here a prediction is implicitly introduced: if γ_s and ψ_k are found somewhere together, then γ_s will overlie ψ_k.

INTRODUCTION OF THE EQUIVALENCE RELATION

Many features not included into a stratifying succession may appear useful for correlation. It may turn out, for example, that detaility of dismembering, which is ensured by a chosen succession is excessive, that every map unit embraces several of its elements at one and the same time. In this case even those features will be useful which give boundaries crossing these features of the succession but not crossing the rest features. Another case is also possible. Let a feature α_i be not stratifying in reference to all the features of the succession except β_j. But even such a feature may turn out as useful for example for the establishment of stratigraphic relations between two bodies. Availability of a feature α_i in one of them and of β_j in another one is the only information about those bodies. Only features not stratifying in reference to all the feature of the succession will bring no use.

To use the features left aside when building a stratifying succession let us introduce the equivalence relation.

Let the relation "to be equivalent" in general mean "to have the same relation." Depending on what exactly the feature has the same relation we discriminate particular kinds of equivalence.

One-sided Equivalence

If α_i is situated under γ_s and besides does not have relations R_1 or R_3 with none of the members of a stratifying succession, overlying γ_s, and β_j also underlies γ_s and besides does not have relations R_1 or R_3 with no one member of the succession over γ_s, then α_i is one-sided equivalent to β_j in relation to γ_s, not depending on the immediate stratigraphic relations between them.

Two-sided Equivalence

If α_i and β_j are in one-sided equivalence with one another in relation to γ_s and are situated under it, and, besides, they are in one-sided equivalence with one another in relation to φ_k and are situated over it, and if, besides, γ_s overlies φ_k, we'll say that α_i is equivalent to β_j in relation to γ_s, or α_i is equivalent to β_j in the interval $\gamma_s - \varphi_k$ not depending on what immediate stratigraphic relations are between them.

Full Equivalence

By a feature fully equivalent to the element of a stratifying succession γ_s we'll call such a feature, which is in one-sided equivalence to γ_s in relation to the element of the succession immediately overlying it and in relation to immediately underlying one, and is situated under an overlying feature and over an underlying one.

It is easy to make sure that all the three introduced relations possess the reflexivity properties, symmetry and transitivity. Consequently, they belong to the group of equivalence.

The usage of the equivalence relation permits to pass from correlation with the help of individual features to correlation with the help of a complex of equivalent features.

Let us construct a rectangular matrix. Its lines are marked by symbols of features of a stratifying succession situated in a stratigraphic succession in the upward direction. In accordance with the transitivity properties of the order relation, every member of the succession has the relation R_2 with any other member situated over and the relation R_1 with any member situated under it. In the matrix columns we'll put symbols of the features not included into a stratifying succession.

Cells of a matrix we fill with symbols of the relations from the earlier built square matrix. One and the same pair of features may satisfy the requirement of two-sided equivalence in relation to several not coinciding intervals at one and the same time. To choose the narrowest among them is in our interest.

We have the right to broaden the interval of equivalence arbitrarily: if α_i and β_j are situated under γ_s, they will be all the more under δ_t situated in a succession over γ_s. Similar reasoning may be made about low boundary of equivalence interval. The right to make such transformations we get if the transitivity property extends to the relations of features not included into a succession to the features of the succession itself. To narrow the equivalence interval arbitrarily is prohibited.

When necessary we can unite all the members of the succession itself, situated in the interval between γ_s and φ_k by the equivalence relation in reference to $\gamma_s - \varphi_k$ both one with another and with other features not included into a succession but equivalent to one another in the same interval. In the same way the one-sided equivalence can be deformed. The upper boundary of the one-sided equivalence may be arbitrarily lifted and the lower one moved below. The reverse is not allowed. Such manners will be necessary in less detailed but more wide correlation.

It is obvious that the concept of "the interval of equivalence" represents a logico-mathematical precising of a traditional concept of "a stratigraphic range." For the proposed concept both a set of observations from which the concept is inferred and the procedure of inference are defined unambiguously.

All the features from the complex included into the equivalence interval can be considered as connected by a logical conjunction "vel." All the equivalent features are interchangeable: inference about correlation made in accordance with α_i is the same if we change α_i by β_j equivalent to it. This circumstance becomes extremely important if neither α_i nor β_j are distributed everywhere: the possibility of finding any one among all the features is much greater than the possibility to find every one of them individually. By broadening the equivalence interval we lessen detailing of dismembering but at the same time greatly increase a number of equivalent features and the possibility to find one of them, and, therefore, the correlation fitness for the work.

Let establish all the possible equivalences and record them in the memory of a computer. Let begin with the establishment of the narrowest intervals of the equivalence. It will permit us to reduce computering operations because the features equivalent to one another in a narrower interval may be immediately included into a list of equivalent ones in a broader interval embracing the first. The problem of the equivalence intervals $\gamma_s - \varphi_k$ establishment consists in finding such columns in rectangular matrix, where symbols R_1 are present in the lines corresponding to γ_s, symbols R_1 or vacant cells are in the lines over γ_s, a symbol R_1 is in the line corresponding to φ_k and symbols R_2 or vacant cells are in the lines under φ_k.

When establishing a stratigraphic succession of the features belonging to different uncrossing and not included into one another intervals we use the transitivity property only if a connecting element is a member of a stratifying succession. This important condition allows to avoid contradictoriness in correlation. It should be stressed that from the conditions α over β, and β over γ we shall infer that α is over γ only when β is a member of a stratifying succession.

If from the fact that α and β features belong to successive intervals follows that α is over β and at the same time between α and β were immediately observed the relations "α is under β" or "α is not a feature stratifying in reference to β," we shall broaden stratigraphic ranges of α and β features till they cross with one another. Range of a more useful feature we leave without changes, broadening only range of a less useful one. In accordance with earlier accepted criterion the most useful will be a feature with the largest sum $R_1 - R_2$ in the corresponding to it line of a square matrix.

USAGE OF A STRATIFYING SUCCESSION AND STRATIGRAPHIC RANGES FOR CORRELATION

Suppose we have a problem to trace throughout the area units discriminated in a stratotype section. Available stratifying succession and stratigraphic ranges we'll transform in such a way that inside one unit of a stratotype will not be present several members of a stratifying succession. For this, stratigraphic ranges are broadened to the maximum, but they must remain within the limits of given bodies.

For given bodies we assume that boundaries do not cross the boundaries of stratigraphic ranges. Because distribution of stratigraphic ranges may be widely traced throughout the area in different sections, plurality of the correlation problem solutions is greatly reduced. If in some section there appeared a possibility to discriminate the same equivalence interval, which contained only one body in a stratotype and in the section under analysis also only one body is present, then we attain uniqueness of the solution.

When reducing a number of variants, let always use the condition of Voronin-Rodionov. Taking into consideration that only identical in their composition bodies can be correlated, we may even more broaden the equivalence intervals. One interval in a stratotype may embrace several bodies unless no two identical in composition bodies are among them. Even if in the correlated sections tens of bodies will occur in one and the same equivalence interval, the variant of correlation will be unique because to one body of class α in a stratotype will correspond only one body of the same class in a section being compared.

CORRECTION OF A STRATIFYING SUCCESSION AND STRATIGRAPHIC RANGES ON THE BASIS OF NEW FACTUAL MATERIAL

When using a stratifying succession and stratigraphic ranges, additional factual material about the features interrelations will be received because new sections are being described. This new material can be entered into a rectangular matrix immediately after its receiving. It can sufficiently change a stratifying succession.

First of all it is necessary to check whether new data confirm earlier inferences about a succession of stratifying features. For this aim to a rectangular we'll add columns corresponding to every feature of a succession. According to the transitivity of order relation in a given succession to the right and higher diagonal of this part of a matrix, there will everywhere be symbols R_1 in cells; to the left and lower diagonal there will be symbols R_2. Let begin to fill the matrix with new observations. If symbols of the relations in every cell will double those, which are present already, that will mean that new factual material confirms the succession built earlier. But if we have to put a symbol different from that which was in it, it will mean that the features corresponding to this cell are not stratifying in relation to one another. They must be taken away from the succession.

Instead of the features taken away, the features fully equivalent to them may be put into the succession. If those do not exist we may connect the divided parts without replacing excluded elements. It is necessary in this case, that the nearest elements of the divided parts are connected by the observed order relations. If such a connection does not exist, the divided parts of a succession must be reduced till their ends are connected by the observed relation R_1 or R_2.

Besides the observations about interrelations between features of a stratifying succession itself new factual material will bring new information about relations of the features not included into a succession with the features of a succession.

First of all it is necessary to check whether an introduction of new members of a succession affected already built stratigraphic ranges. For this aim we have to test on the basis of initial square matrix whether in a line corresponding to a new member are not symbols R_1 or R_3 where this line crosses the columns corresponding to the features of all the ranges lying higher, and symbols R_2 or R_3 where this line crosses the columns corresponding to the features of all the ranges lying lower. In case such columns are found, it is necessary to broaden stratigraphic ranges of corresponding to them features. A new member of a succession must be included into the range: for the ranges lying lower the upper boundary should be lifted higher than a new member is located; for those ranges which lie higher the lower boundary should be moved below.

Then we have to test whether new material has changed available before stratigraphic ranges. For this aim for every feature fixed in new factual material it is necessary to test by rectangular matrix whether symbols R_2 or R_3 have not appeared in cells of a column of this feature higher than the upper boundary of the interval and symbols R_1 or R_3 in cells lower than the lower boundary. If there are cells with such symbols we broaden ranges till those cells are included inside the range.

Let also exclude contradictions if they appeared: from α and β features belonging to successive ranges follows that α is over β while new material brought observations "α is under β" or "α is not stratifying in relation to β." A procedure of excluding such contradictions is described earlier.

It is possible that new material will bring observations about the features occurring over the uppermost member of a stratifying succession with which no one member of a succession has relations R_1. An opportunity appears to complete a succession upwards. For this we construct a square matrix. Its lines and columns will be all the features found over the uppermost member of a succession. We make full sorting "everyone with everyone" according to the data of immediate observations and build a fragment of a stratifying succession according to a technique described earlier. The result we add from the top to a succession available before. The same is done with the features situated under the lowest member of the old succession.

Perhaps we shall not be satisfied by the detailing of a succession in any one of its parts. We then suppose that new material will help us to detail the succession available. For this we have to take all the features equivalent to one another in the interval interesting to us including the features of a succession itself. Then we have to make sorting "every one with every one" in a square matrix. Later according to a square matrix we build a part of a succession and place it into a breach formed where the equivalence interval was placed. In the same way we can detail any part of a succession or any number of such parts separately.

CONCLUSION

The technique described is a logico-mathematical precising of practical methods of stratigraphic constructions. This makes it different from a majority of other mathematical methods. From traditional geological methods the algorithm differs by uniqueness, unambiguous conclusions. One may speak of proper stratigraphic mathematical calculus, the procedures providing a success of correlation without dependence on the distance of correlation and nature of the features used, are summed up. When processing lithological, geochemical, and geophysical features according to the proposed technique, a detailed local correlation can be fulfilled; when processing paleonthological features undetailed interegional planetary correlation can take place. By introducing later on unambiguous mathematical methods of separating already built stratifying succession and its ranges into parts, we may achieve logico-mathematical precising of our ideas about zones, regional and planetary stages, systems, and groups.

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